MADHYANCHAL
PROFESSIONAL UNIVERSITY

Draft Rules & Syllabus
for the

Master of Science in Statistics

(M. Sc. Stats.) Course

MADHYANCHAL PROFESSIONAL UNIVERSITY

DEPARTMENT OF MATHEMATICS

Scheme for M.Sc. CBCS Course

Semester I

S.No.	Subject Code	Subject Name & Title	Maximum Marks Allotted								Hours per week			Total Credits	Remarks
			Theory				Practical								
			End Sem	Mid Sem. MST	Quiz, Assignment	Total Marks	Lab Work	Assignment /Quiz/Term paper	End Sem	Total Marks	L	T	P		One credit refers to one hour teaching in theory, Tutorial
1	M. Sc 101	Mathematical Analysis and Linear Algebra	60	20	20	100	-	-	-	-	3	1		4	
2	M. Sc 102	Probability Theory (PT)	60	20	20	100	-	-	-	-	3	1		4	
3	M. Sc 103	Distribution Theory	60	20	20	100	-	-	-	-	3	1		4	
4	M. Sc 104	Theory of Estimation (ET)	60	20	20	100	-	-	-	-	3	1		4	
5	M. Sc 101 P	C++ Programming					40	20	40	100			4	2	
6	M. Sc 102 P	Linear Algebra, Distribution Theory and Theory of Estimation					40	20	40	100			4	2	
	Total		240	80	80	400	80	40	80	200	12	4	8	20	600

Semester II

S.No.	Subject Code	Subject Name & Title	Maximum Marks Allotted								Hours per week			Total Credits	Remarks
			Theory				Practical								
			End Sem	Mid Sem. MST	Quiz, Assignment	Total Marks	Lab Work	Assignment /Quiz/Term paper	End Sem	Total Marks	L	T	P		One credit refers to one hour teaching in theory, Tutorial
1	M. Sc 201	Sampling Techniques (ST)	60	20	20	100	-	-	-	-	3	1		4	
2	M. Sc 202	Stochastic Processes (SP)	60	20	20	100	-	-	-	-	3	1		4	
3	M. Sc 203	Linear Models and Design of Experiments (LM and DOE)	60	20	20	100	-	-	-	-	3	1		4	
4	M. Sc 204	Multivariate Analysis	60	20	20	100	-	-	-	-	3	1		4	
5	M. Sc 201 P	Sampling Techniques and Stochastic Processes					40	20	40	100			4	2	
6	M. Sc 202 P	Linear Models, Design of Experiments and Multivariate Analysis					40	20	40	100			4	2	
	Total		240	80	80	400	80	40	80	200	12	4	8	20	600

Chapter II Syllabus

Paper I - Mathematical Analysis and Linear Algebra (MA and LA)

UNIT–I

Functions of Bounded Variation (BV). Total variation and its additive Property.

Functions of BV expressd as the difference of increasing functions.

Riemann-Steiltjes (R-S) Integral and its linear properties. Integration by parts. Euler's summation. Riemann's condition. Integrators of BV. Statements of necessary and sufficient conditions of R-S integral. Differentiation under the integral sign. Interchanging the order of integration.

UNIT–II

Complex derivatives. Cauchy-Riemann equations. Analytic functions. Statements of Cauchy theorem and integral formula. Power, Taylor's and Laurent's series. Zeroes and poles. Statement of Cauchy residue theorem. Cantour integration. Evaluation of real valued integrals by means of residues.

Functions of several variables-concepts of limit, continuity, directional derivatives, partial derivatives, total derivative, extreme and saddle points with examples. Taylor's expansion. Multiple Integration. Application of Jacobians in the evaluation of multiple integrals.

UNIT – III

Vector spaces with an inner product, Gram-Schmidt orthogonolization process, orthonormal basis and orthogonal projection of a vector.

Moore-Penrose and generalized inverses and their properties. Solution of matrix equations. Sufficient conditions for the existence of homogeneous and non-homogeneous linear equations.

UNIT – IV

Characteristic roots and vectors, Caley-Hamilton theorem, algebraic and geometric multiplicity of a characteristic root and spectral decomposition of a real symmetric matrix.

Real quadratic forms (QFs), reduction and classification of QFs, index and signature. Simultaneous reduction of two QFs. Extreme form of a QF. Cauchy-Schwartz and Hadamard inequalities for matrices.

REFERENCES

1. Apostol,T.M. (1985) : Mathematical Analysis, Narosa, Indian Ed.
2. Malik,S.C. (1984) : Mathematical Analysis, Wiley – Eastern.
3. Rudin,W. (1976) : Principles of Mathematical Analysis, McGraw Hill.
4. Graybill,F.A. (1983) : Matrices with applications in statistics, 2nd ed, Wadsworth.
5. Rao,C.R. (1973) : Linear Statistical inference and its applications, 2nd Ed, John Wiley & Sons Inc.
6. Searle,S.R. (1982) : Matrix algebra useful for statistics, John Wiley and Sons Inc.
7. Rao,C.R., Mithra,S.K. (1971) : Generalised inverse of matrices and its applications, John Wiley & Sons Inc.
8. Rao, A.R. and Bhimasankaram, P(1992) : Linear algebra, Tata – McGrawhill Publishing Co. Ltd.

Paper II - Probability Theory (PT)

UNIT – I

Classes of sets, fields, sigma-fields, minimal sigma-fields, Borel sigma-fields in R, Measure, Probability Measure, Properties of a Measure, Caratheodory extension theorem (Statement only), measurable function, random variables, distribution function and its properties, expectation, statements and applications of monotone convergence theorem, Foatou's lemma, dominated convergence theorem.

UNIT – II

Expectations of functions of rv's, conditional expectation and conditional variance, their applications. Characteristic function of a random variable and its properties. Inversion theorem, uniqueness theorem (Functions which cannot be Characteristic functions). Levy's continuity theorem (Statement only). Chebychev, Markov, Cauchy- Schwartz, Jenson, Liapunov, Holder's and Minkowsky's inequalities.

UNIT – III

Sequence of Random variables, convergence in Probability, convergence in distribution, almost sure convergence, convergence in quadratic mean and their interrelationships, Slutskey's theorem, Borel-Cantelli lemma Borel 0-1 law, Kolmogorov 0-1 law (Glevenko – Cantelli Lemma -Statement only).

UNIT – IV

Law of large numbers, Weak law of large numbers, Bernoulli and Khintchen's WLLN's, Kolomogorov Inequality, Kolmogorov SLLN for independent random variables and statement only for i.i.d. case, statements of three series theorem.

Central Limit theorems : Demoviere - Laplace CLT, Lindberg-Levy CLT, Liapounou' CLT, Statement of Lindberg-Feller CLT, simple applications, statement of Cramer-Wald theorem, Asymptotic distribution of sample quantiles.

REFERENCES

1. Ash Robert (1972) : Real analysis and Probability, Academic Press
2. Bhat,B.R. : Modern probability Theory, 3rd Edition, New Age India
3. Rohatgi,V.K. : Introduction to Probability Theory and Mathematical Statistics
4. Milton and Arnold – Introduction to probability and Statistics (4th Edition)- TMH publication.

ADDITIONAL REFERENCES

1. Kingman,J.F.C. and Taylor, S.J. (1966) : Introduction of measure and probability, Cambridge University press
2. Basu,A.K. : Probability and Measure,Narosa (PHI)
3. W.Feller : An Introduction to Probability theory and its Applications Vol I and II, John Wiely.

Paper III - Distribution Theory (DT)

UNIT – I

Normal, Lognormal, Weibull, Pareto and Cauchy distributions and their properties. Joint, Marginal and conditional pmf's and pdf's.

UNIT – II

Families of Distributions: Power series distributions, Exponential families of distributions. Functions of Random variables and their distributions (including transformation of rv's). Bivariate Normal, Bivariate Exponential (Marshall and Olkins form), Compound Binomial - Poisson, Gamma(□,□). Truncated (Binomial, Poisson, Normal and Lognormal) and mixture distributions -Definition and examples.

UNIT – III

Sampling Distributions of sample mean and variance, independence of $\bar{X}$ and s^2. Central and Non-central χ^2, t and F distributions.

UNIT – IV

Distributions of quadratic forms under normality and related distribution theory. Order statistics, their distributions and properties. Joint and marginal distributions of order statistics and Distribution of Range. Extreme values and their asymptotic distributions (statements only) with applications.

REFERENCES

1. Rohatgi,V.K.(1984) : An introduction to probability theory and mathematical Statistics, Wiley Eastern.
2. Rao,C.R. (1972) : Linear Statistical Inference and its applications, 2/e, Wiley Eastern
3. Milton and Arnold – Introduction to probability and Statistics (4^{th} Edition)- TMH publication.

ADDITIONAL REFERENCES

1. Pittman,J. (1993) : Probability, Narosa Publishing House
2. Johnson,S. and Kotz,(1972) : Distributions in Statistics,Vol. I,II and III, Houghton and Miffin.
3. Cramer, H. (1946) : Mathematical methods of statistics, Princeton.
4. Dudewicz,E.J., and Mishra, S.N. (1988) : Modern Mathematical statistics, Wiley International Students edition.

Paper IV - Theory of Estimation (ET)

UNIT – I

Point Estimation Vs. Interval Estimation, Advantages, Sampling distribution, Likelihood function, exponential family of distribution.

Desirable properties of a good estimator: Unbiasedness, consistency, efficiency and sufficiency - examples. Neyman factorization theorem (Proof in the discrete case only), examples. UMVU estimation, Rao-Blackwell theorem, Fisher Information, Cramer- Rao inequality and Bhattacharya bounds.

UNIT II

Completeness and Lehmann-Scheffe theorem. Median and modal unbiased estimation. Estimation of bias and standard deviation of point estimation by the Jackknife, the bootstrap methods with examples.

UNIT III

Methods of estimation, method of moments and maximum likelihood method, examples. Properties of MLE. Consistency and asymptotic normality of the consistent solutions of likelihood equations. Definition of CAN and BAN, estimation and their properties, examples.

UNIT IV

Concept of U statistics and examples. Statement of Asymptotic distributions of U – statistics. Interval estimation, confidence level CI using pivots and shortest length CI. Confidence intervals for the parameters for Normal, Exponential, Binomial and Poisson Distributions. Confidence Intervals for quintiles. Concept of tolerance limits and examples.

REFERENCES

1. Goon, Gupta and Das Gupta : Outlines of Statistics, Vol. 2, World Press, Calcutta.
2. Kale, B.K. (1999): A first course on parametric inference, Narosa publishing house.
3. Rohatgi, V.K.: An introduction to Probability theory and mathematical statistics, Wiley Eastern.

ADDITIONAL REFERENCES

1. Rao, C.R.: Linear Statistical Inference and its applications, John Wiley
2. Gray and Schucany : Generalized Jackknife; Marcel Decker
3. Bradely Efron and Robert J. Tibshirani : An Introduction to the Bootstrap, Chapmen and Hall.
4. Lehman, E.L. (1983) : Theory of point estimation, John Wiley
5. Gray, Schncory and Watkins : Generalized Jacknife, Dovenpul

Paper V - Practical (C++ Programming)

Concepts to be covered: Principles of Object Oriented Programming, Tokens, Expressions and Control structures. Functions, classes and objects. Constructors and destructors. Operator overloading and type conversions, Inheritance, Pointers, Virtual functions and Polymorphism. Managing console I/O operations. Working with files. Object oriented system development. Templates and exception handling.

List of Practicals:

1) a) Factorial of a number b) Fibonacci series generation
2) a) Pascal triangle b) Pyramid of digits
3) Evaluation of a) e^x b) sin x c) cos x using series expansion.
4) Find a) mean b) variance c) standard deviation and d) coefficient of variation for a given set of data.
5) a) Finding correlation coefficient and b) fitting straight line regression and parabolic regression curve.
6) Sorting numbers by bubble sort and finding median and mode of the data.

7) Write a program for preparation of frequency tables using functions and computing mean, median, mode, variance and standard deviation of the frequency distribution.

8) Write a class to a) create a vector b) modify the values of a given element c) to multiply by a scalar value and d) display the vector in the form of a row vector. Write a main program to test your class.

9) Display and addition of complex numbers and vectors by creating a complex and vector class, respectively.

10) Matrix addition, subtraction and multiplication of confirmable matrices by operator over loading.

11) Concatenation of two strings using operator overloading.

12) File opening, writing records, reading records and updating a file, prepare merit list of students for an entrance examination marks from a file. Write the merit list on some other file and display the same.

13) Define a base class 'B' containing one private data member 'a' and public data member 'b' and three public member functions get_ab ( ), get_ a (Void), show_a (void). Derive a class 'D' from the class 'B', 'D' should contain one private data member 'C' and two member functions Mul (void) and Display (void). Define a main program in which create an object for the class and test all the four member functions.

14) Generation of uniform random numbers using virtual functions.

15) Fitting of distributions - Binomial, Poisson and Negative binomial based on relation between mean and variance.

16) Solution to simultaneous equations by Gauss - Siedal method.

REFERENCES

1. Balagurusamy, E.(1995) : Object – oriented Programming with C++, Tata Mc Graw Hill
2. K. R. Venugopal and others (2005) : Mastering C++, Tata Mc Graw Hill
3. Strousstroup, B.(1991) : The C++ Programming Language, 2nd edition, Addison-Wesley.

Paper VI PRACTICALS IN LINEAR ALGEBRA, DISTRIBUTION THEORY AND ESTIMATION

LINEAR ALGEBRA

1. Inverse of a matrix by partition method
2. Solutions of linear equations by sweep-out method
3. Solutions of linear equations by Doolittle Method
4. Computation of Moore-Penrose inverse by Penrose method
5. Computation of generalized inverse of a matrix.
6. Formation of characteristic equation by using traces of successive powers
7. Spectral decomposition of a square matrix of third order
8. Simultaneous reduction of a pair of quadratic forms to diagonal and canonical forms.
9. Finding orthonormal basis by Gram – Schmidt process.

DISTRIBUTION THEORY

1. Fitting an appropriate distribution (Binomial, Poisson, Negative Binomial)
2. Fitting of Normal and Exponential Distributions
3. Fitting of Cauchy distributions
4. Fitting of Pareto distribution.
5. Discrete Bivariate distributions.

ESTIMATION

1. Computation of Jackknife estimates
2. Computation of Boot-strap estimates
3. MLE by Scoring method
4. Confidence limits for parameters of normal population
5. Large sample confidence limits in case of Binomial, Poisson, Exponential distributions.

UNIT I

Review of SRSWR/WOR, Stratified random sampling and Systematic Sampling. Unequal probability Sampling: ppswr/wor methods (including Lahiri's scheme) and related estimators of a finite population mean. Horowitz – Thompson, Hansen – Horowitz and Yates and Grundy estimators for population mean/total and their variances.

UNIT – II

Ratio Method Estimation: Concept of ratio estimators, Ratio estimators in SRS, their bias, variance/MSE. Ratio estimator in Stratified random sampling – Separate and combined estimators, their variances/MSE.

Regression method of estimation: Concept, Regression estimators in SRS with pre – assigned value of regression coefficient (Difference Estimator) and estimated value of regression coefficient, their bias, variance/MSE, Regression estimators in Stratified Random sampling – Separate and combined regression estimators, their variance/ MSE.

UNIT – III

Cluster Sampling: Cluster sampling with clusters of equal sizes, estimator of mean per unit, its variance in terms of intracluster correlation, and determination of optimum sample and cluster sizes for a given cost. Cluster sampling with clusters of unequal sizes, estimator - population mean its variance/MSE.

Sub sampling (Two – Stage only): Equal first stage units – Estimator of population mean, variance/MSE, estimator of variance. Determination of optimal sample size for a given cost. Unequal first stage units – estimator of the population mean and its variance/MSE.

UNIT – IV

Non – Sampling errors: Sources and treatment of non-sampling errors. Non – sampling bias and variance.

Randomized Response Techniques (for dichotomous populations only): Warner's model, unrelated question model.

Small area estimation : Preliminaries, Concepts of Direct Estimators, Synthetic estimators and Composite estimators.

REFERENCES

1. Parimal Mukhopadhyay (1998) : Theory and methods of Survey sampling, Prentice – Hall of India, New Delhi.

2. Murthy, M.N. (1967): Sampling Theory and methods, Statistical Publishing Society, Calcutta.

ADDITIONAL REFERENCES

1. Des Raj (1976) : Sampling Theory, Tata McGraw Hill, New Delhi.
2. Sukhatme etal (1984) : Sampling Survey methods and its applications, Indian society of Agricultural Statistics.
3. Cochran, W.C. (1977) : Sampling Techniques, Third Edition, Wiley Eastern.

Paper II

UNIT I

Introduction to stochastic processes; classification of stochastic process according to state-space and time-domain. Finite and countable state Markov chains; time-homogeneity; Chapman-Kolmogorov equations; marginal distribution and finite – dimensional distribution; classification of states of a Markov chain – recurrent, positive recurrent, null - recurrent and transient states. Period of a state.

UNIT – II

Canonical form of transition probability matrix of a Markov chain. Fundamental matrix; probabilities of absorption from transient states into recurrent classes, in a finite Markov Chain; mean time for absorption. Ergodic state and ergodic chain. Stationary distribution of a Markov chain. Existence and evaluation of stationary distribution. Random walk and gambler's ruin problem.

UNIT – III

Discrete state-space, continuous time Markov Processes – Kolmogorov difference - differential equations. Poisson process and its properties. Birth and Death Process, application in queuing. Pure Birth and pure Death processes.Weiner process as limit of random walk. First passage time of the process.

UNIT – IV

Renewal process, elementary renewal theorem and its applications. Statement and uses of Key – renewal theorem. Residual life time. Branching process – Galton-Watson branching process, mean and variance of size of n^{th} generation; probability of ultimate extinction of a branching process – fundamental theorem of Branching process – Examples.

REFERENCES

1. Medhi,J. (1982) : Stochastic Processes – Wiley Eastern
2. Karlin, S. and Taylor, H.M. (1975): A First Course in Stochastic Processes, Vol. I, Academic Press.

ADDITIONAL REFERENCES

1. Bhat, B.R. (2000): Stochastic Models: Analysis and applications – New Age International India.
2. Basu, A.K. (2003): Introduction to Stochastic Process, Narosa Publishing House.

PAPER 3 - Linear Models and Design of Experiments (LM & DOE)

UNIT– I (LM)

Formulation of a linear model through examples. Estimability of a linear parametric function. Gauss-Markov linear model, BLUE for linear functions of parameters, relationship between BLUE's and linear Zero-functions. Gauss-Markov theorem, Aitkin's generalized least squares, Concept of Multi-collinearity.

UNIT– II

Simple linear regression, examining the regression equation, Lack of fit and pure error. Analysis of Multiple regression models. Estimation and testing of regression parameters, sub-hypothesis. Introduction of residuals, overall plot, time sequence plot, plot against Yi, Predictor variables X_{ij}, Serial correlation among the residual outliers. The use of dummy variables in multiple regression, Polynomial regressions –use of orthogonal polynomials. Derivation of Multiple and Partial correlations, tests of hypothesis on correlation parameters.

UNIT– III (DOE)

Analysis of Covariance:One-way and Two-way classifications.

Factorial experiments: Estimation of Main effects, interaction and analysis of 2^k, factorial experiment in general with particular reference to k = 2, 3 and 4 and 3^2 factorial experiment. Multiple Comparisons: Fishers least significance difference (LSD) and Duncan's Multiple Range test (DMR test).

UNIT – IV

Total and Partial Confounding in case of 2^3, 2^4 and 3^2 factorial designs. Concept of balanced partial confounding.

Fractional replications of factorial designs: One half replications of 2^3 and 2^4 factorial designs, one-quarter replications of 2^5 and 2^6 factorial designs. Resolution of a design. Split – Plot design.

REFERENCES

1. Searles S.R.(1971):Linear statistical Models.
2. Draper and Smith: Applied Regression Analysis
3. Montogomery,D.C.:Design and Analysis of Experiments, John Wiley
4. Giri, N.C.:Analysis of Variance

ADDITIONAL REFERENCES

1. Kshirasagar A.M.(1972): A course in Linear Models.
2. Graybill F.A(1966): An introduction to linear statistical models- Vol.I
3. Gultman (1982): Linear Models - An Introduction.
4. Rao A.R and Bhimsankaram P: Linear Algebra – Hindustan Agency.
5. Kempthrone: Design and Analysis of Experiments.
6. Cochran and Cox: Experimental Designs.

PAPER 4 - Multivariate Analysis (MVA)

UNIT – I

Multinomial distribution Multivariate normal distribution, marginal, conditional distributions. Independence of multivariate vectors. Random sampling from a multivariate normal distribution. Maximum likelihood estimators of parameters. Distribution of sample mean vector. Independence of sample mean vector and variance-covariance matrix.

UNIT – II

Wishart matrix – its distribution and properties. Distribution of sample generalized variance. Null distribution of simple correlation coefficients. Null distribution of partial and multiple correlation coefficients. Distribution of sample regression coefficients. Application in testing and interval estimation.

UNIT – III

Null distribution of Hotelling's T^2 statistic. Application in tests on mean vector for one and more multivariate normal populations and also on equality of the components of a mean vector in a multivariate normal population.

Mahalanobi's D^2 statistic. Wilk's □ - criterion and statement of its their properties with simple applications. Classification and discrimination procedures for discrimination between two multivariate normal populations – sample discriminant function, tests associated with discriminant functions, probabilities of misclassification and their estimation, classification into two multivariate normal populations with equal covariance matrices.

UNIT – IV

Principal components, Dimension reduction, graphical of Principal Components, canonical variables and canonical correlation – definition, use, estimation and computation.Concepts of cluster analysis and multi – dimensional scaling. Introduction to Factor analysis, orthogonal factor model.

REFERENCES

1. Anderson, T.W. (1983) : An Introduction to multivariate statistical analysis, 2nd Edition, Wiley.
2. Kshirasagar, A.M. (1972) : Multivariate Analysis, Marcel Decker.
3. Johnson, R.A.W.: Applied Multivariate Analysis.

ADDITIONAL REFERENCES

1. Giri, N.C. (1977): Multivariate statistical inference, Academic Press
2. Morrison, D.F. (1976): Multivariate Statistical Methods, 2nd Edition, McGraw Hill
3. Muirhead, R.. (1982) : Aspects of multivariate statistical theory, J. Wiley.

PAPER 5 Practical (ST and SP)

PRACTICALS IN SAMPLING TECHNIQUES AND STOCHASTIC PROCESSES

SAMPLING TECHNIQUES

1. PPS sampling with and without replacements.
2. Ratio estimators in SRS , comparison with SRS
3. Separate and combined ratio estimators, Comparison.
4. Regression estimators in SRS, Comparison with SRS and Ratio estimators
5. Separate and combined Regression estimators, Comparison.
6. Cluster sampling with equal cluster sizes.
7. Sub sampling (Two–stage sampling) with equal first stage units.

STOCHASTIC PROCESSES

1. Formulation of problems as Markov chain models
2. Computation of finite dimensional and marginal distributions; higher dimensional transition probabilities.
3. Classification of states, identification of recurrent classes and reduction to canonical form of t.p.m.
4. Probabilities of absorption into recurrent classes (from transient states)
5. Computation of stationary distribution (unique case)
6. Computation of stationary distribution (non-unique case)
7. M|M|1 queue – operating characteristics
8. Mean and variance of n^{th} generation size and probability of extinction of Branching processes.

Paper VI Practical (LM & DOE and MVA)

PRACTICALS IN LINEAR MODELS, DESIGNS OF EXPERIMENTS AND MULTIVARIATE ANALYSIS

LINEAR MODELS AND DESIGNS OF EXPERIMENTS

1. Computation of BLUE and testing their parameters.
2. Computation of Pure error and Lack of fit.
3. Computation of residuals and their plots for two and three variables.
4. Computation of Multiple Correlation coefficient
5. Computation of Partial Correlation coefficient
6. Testing of Multiple and Partial Correlation Coefficients.
7. Analysis of 2^3, 2^4 and 3^2 factorial experiments.
8. Analysis of total confounding and partial confounding of 2^3 design.
9. Analysis of one–half fraction of 2^4 design and one–quarter fraction of 2^5 design.
10. Analysis of Split-Plot design.

MULTIVARIATE ANALYSIS

1. MLE of Mean vector and variance covariance Matrix from Normal population.
2. Hotelling's T^2 and Mahalanobi's D^2.
3. Computation of Principal components.
4. Classification between two normal populations by discriminantanalysis.
5. Cluster analysis.Computation of Canonical variables and correlation.